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Evaluating the Impact of Optical Axis Stability on Exoplanet Detection

Dong-Jie Tan, Jia-Cheng Liu, Zi Zhu, and Niu Liu
School of Astronomy and Space Science, Key Laboratory of Modern Astronomy and Astrophysics (Ministry of Education), Nanjing University, 163 Xianlin Avenue, 210023 Nanjing, China; jcliu@nju.edu.cn
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Abstract

For detecting exoplanets with high precision, using the angular distance between the two stars to detect the periodic motion of the star will be a better choice. This approach can avoid importing the position error of the reference catalog in the process that uses a traditional photographic plate to derive the star position suffers. At the precision level of microarcseconds, the error caused by optical axis deviation is not negligible. In this paper, we evaluate the impact of the stability of the optical axis on the relative angular distance measurement from the aspects of theoretical analysis and numerical simulation. When the angular distance error limit of 1 microarcsecond is given, the upper limit of optical axis deviation is estimated to be 68 milliarcsecond. In addition, when limiting the deviation of the optical axis, we give the corresponding error allowance of angular distance measurement. Moreover, we also discuss the way to resolve the problem of CCD distortion and focal length change on the measurement of angular distance. The work in this paper is of guiding significance to the design of a telescope.

Key words: instrumentation: high angular resolution – methods: numerical – planets and satellites: detection

1. Introduction

In the past two decades, searching for exoplanet systems and discovering habitable planets have been hot topics. During this period, a series of detection approaches have been developed. Two main detection approaches often mentioned in exoplanet exploration are radial velocity and photometry (Perryman 2000). They have excellent performance in detecting exoplanets, however, they cannot measure the complete orbital parameters and the mass of exoplanets. The Search for Terrestrial Exo-Planet (STEP; Malbet et al. 2012) and the Closeby Habitable Exoplanet Survey (CHES; Jianghui & Wang 2020) employed an approach of relative measurement. This approach combined with the radial velocity and other technologies provides a complete measurement of orbital parameters and mass of the exoplanets that cannot be measured by using radial velocity or photometry exclusively.

Utilizing astrometry to search for exoplanets is an emerging technology and still in its infancy. The advantage is that it can detect the periodic term of star motion caused by the effect of gravity, thus it infers the existence of an exoplanet. For a solar-like star which is 1 pc away from us and surrounded by a planet with the Earth's mass and the semimajor axis of the orbit being 1 au, the allowable measurement error of the position to detect the existence of the planet is 1 μas (Malbet et al. 2012).

In photographic astrometry, the plate constant is obtained from the position of reference stars in the prior catalog, which is used to derive the position of the target star (Kovalevsky & Seidelmann 2004). Positional error in the reference catalog is imported during this process. So, a relative measurement approach (Shu-yu et al. 2018) that measures the angular distance between any reference star and the target star in the field of view (FOV) to detect the periodic motion of the star would be a better choice. It uses the angular distance as a new position parameter. Through this approach, we can avoid the error inherited from the star catalog.

The measurement precision of angular distance between star pairs is limited by several factors. The first one is the limitations of the optical system, including the precision and distortion of the charge-coupled device (CCD; Jin et al. 2013). The second one is the stability of the optical system, where the main factor is stability of the optical axis.

In this paper, we aim to evaluate the impact of optical axis deviation on the measurement of angular distance. We give an introduction to the instruments in Section 2. In Section 3, we perform a theoretical analysis on the impact of the deviation of optical center position of the star and the measurement of angular distance between the star pair. In Section 4, we present the results of numerical simulations and the error distribution of angular distance under different deviations. We set 1 μas as an upper limit to evaluate the impact of the optical axis deviation on the measurement of the target star position. We propose solutions to reduce the impact of lens distortion and focal length change on the measurement of angular distance in Section 5. The summary is given in Section 6.

2. Instrument

CHES (Jianghui & Wang 2020) is a space satellite project with high accuracy ($\sim$1 μas) for exoplanet detection. It aims to directly detect and study Earth-like planets in the vicinity of the solar system (within 10 pc) and achieve narrow-angle measurement at the microsecond level. The aperture of the telescope used is 1.2 m. The FOV is $0^\circ\!.44 \times 0^\circ\!.44$ which meets the observation requirements (at least three reference stars enter the field angle of CHES). The optical structure is a coaxial reflection type, with high imaging quality and low distortion, and a three-mirror anastigmatic structure is used to increase the field angle. The designed single measurement uncertainty is better than 1 μas, and the observational uncertainty can be improved by an order of magnitude if the number of repeated observations reaches 50 times. Assuming that the reference star is a distant single star and the target star is a close star, the perturbation of the planet with respect to the target star can be detected by small changes in the angular distances between the reference stars and the target star with an accuracy of microseconds.

3. Impact of the Optical Axis Deviation from Theoretical Analysis

In this section, we illustrate how the deviation of optical center position impacts the measurement of angular distance of the star pair. First, we define the direction in which the optical axis of the telescope should point. To reduce the variation of the FOV caused by the different pointings of each observation, it is suggested that the optical axis should be pointed at the middle position of the target star's proper motion on the celestial sphere during the observation period. But in this section, we do not restrict the optical axis to the target star, which means that we do not restrict one of the stars to be near the center of light. We analyze the impact of the optical axis deviation on the angular distance between star pairs at any position in the FOV.

Based on the projection theorem, there is a nonlinear relationship between the position of the star on the CCD and that on the celestial sphere. This relationship depends on the projected point which is determined by the optical axis, which means that even a tiny change in the direction of the optical axis will cause a nonlinear change in the position of the star on the CCD. This corresponding relation is illustrated in Figure 1. Stars at different locations on the CCD are affected differently by the deviation of the optical axis, so the angular distance change between star pairs is nonlinear. Suppose that the optical axis is not completely stable, and the angular distance measurement will be biased, which in turns affects the measurement of the periodic motion of the target star and the detection of exoplanets.

3.1. Effect on the Position of a Star

As diagrammed in Figure 2(a), point **O** is the original center of the FOV and point **O'** is the new center of the FOV due to deviation of the optical center. Point **A** is a star in the FOV.

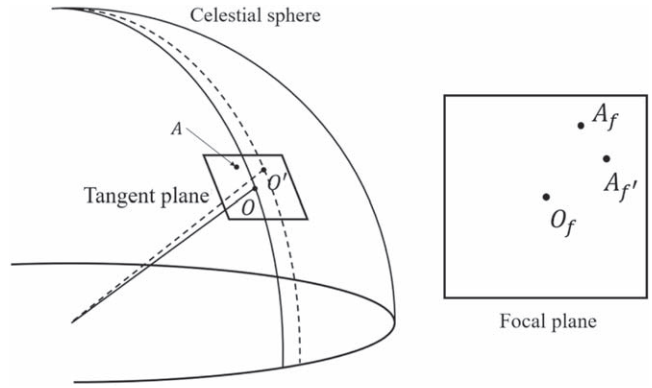

Figure 1. The relationship between the deviation of the optical center and changes in the projected point and star position on the focal plane. In the left panel, point A is a star on the celestial sphere, and point O and point O' are the positions of the optical center before and after the optical axis deviation respectively. In the right panel, point A_f is the star position on the focal plane and point $A_{f'}$ is the new position on the focal plane due to the optical axis deviation. Point O_f is the center of the focal plane.

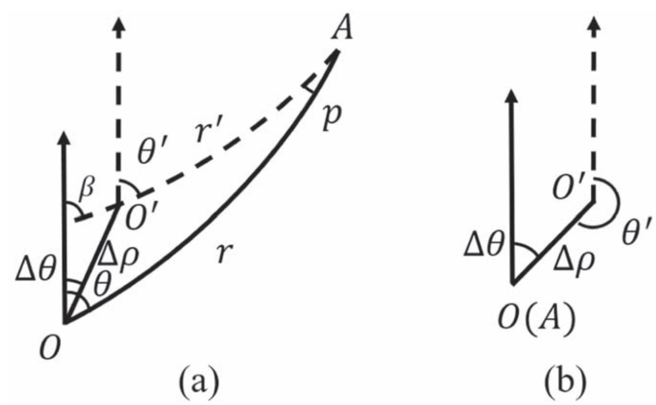

Figure 2. Schematic diagram of the effect of deviation of the optical center position on the celestial sphere. The left panel (a) is the general situation and the right panel (b) is the situation such that the star is at the center of the FOV. Point A is the position of the star. r and r' are the distances between the reference star and the optical center. θ and θ' are the angle between the target star and the latitude line. The points O and O' are the positions of the optical center before and after deviation respectively. $\Delta\rho$ and $\Delta\theta$ are the distance and direction resulting from deviation of the optical center position respectively.

For the spherical triangle **OO'A**, we have

$$\cos r' = \cos\Delta\rho\cos r + \sin\Delta\rho\sin r\cos(\theta - \Delta\theta), \tag{1}$$

$$\frac{\sin r'}{\sin(\theta - \Delta\theta)} = \frac{\sin\Delta\rho}{\sin p}, \tag{2}$$

where r and r' are the distances between the reference star and the optical center, θ and θ' the angle between the target star and the latitude line, and $\Delta\rho$ and $\Delta\theta$ the distance and direction of the new optical center with respect to the old one respectively.

Because the FOV is small (0°.44) and the deviation of optical center position is on the order of milliarcseconds, the two axes in Figure 2 are considered parallel and we can use the small spherical triangle approximation

$$\theta' = \beta = \theta + p. \tag{3}$$

For star **A** at the center **O** of the FOV, we can get the new position of the star from Figure 2(b). When there is a deviation in the optical center position, the new coordinate r' is equal to the value of deviation of the optical center and the new coordinate θ' is the angle in the direction opposite to the deviation angle,

$$r' = \Delta\rho, \tag{4}$$

$$\theta' = \Delta\theta + \pi. \tag{5}$$

In the focal plane, the coordinate is (r_f, θ_f) and the transformation is

$$r_f = \tan r', \tag{6}$$

$$\theta_f = \theta'. \tag{7}$$

3.2. Effect on Angular Distance

For two stars in the FOV, the angular distance l on the focal plane can be written as

$$l = [r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_1 - \theta_2)]^{\frac{1}{2}}, \tag{8}$$

where r_1, r_2, θ_1, θ_2 are the coordinates of the two stars in polar coordinates. When the optical center position has a slight deviation due to a change in the projected point, the position of the two stars on the focal plane will also be changed. We define $\Delta l = |l' - l|$ as the change caused by the deviation of the optical center position.

$$\begin{aligned} &\Delta l = \Delta l(r_1, r_2, \theta_1, \theta_2, r_1', r_2', \theta_1', \theta_2'),\\ &r_1' = r_1'(r_1, \theta_1, \Delta\rho, \Delta\theta), \quad r_2' = r_2'(r_2, \theta_2, \Delta\rho, \Delta\theta),\\ &\theta_1' = \theta_1'(r_1, \theta_1, \Delta\rho, \Delta\theta), \quad \theta_2' = \theta_2'(r_2, \theta_2, \Delta\rho, \Delta\theta), \end{aligned} \tag{9}$$

where l' is the new angular distance and r_1', r_2', θ_1', θ_2' are the new coordinates, and $\Delta\rho$ and $\Delta\theta$ are the distance and direction of the deviation of the optical center position respectively. The angular distance change (Δl) caused by the deviation of the optical center position can be determined by the following formula,

$$\begin{aligned} d\Delta l = &\frac{\partial \Delta l}{\partial r_1} dr_1 + \frac{\partial \Delta l}{\partial r_2} dr_2 + \frac{\partial \Delta l}{\partial \theta_1} d\theta_1\\ &+ \frac{\partial \Delta l}{\partial \theta_2} d\theta_2 + \frac{\partial \Delta l}{\partial \Delta\rho} d\Delta\rho + \frac{\partial \Delta l}{\partial \Delta\theta} d\Delta\theta, \end{aligned} \tag{10}$$

$$\begin{aligned} \frac{\partial \Delta l}{\partial \Delta\rho} = &\frac{\partial \Delta l}{\partial r_1'} \frac{\partial r_1'}{\partial \Delta\rho} + \frac{\partial \Delta l}{\partial r_2'} \frac{\partial r_2'}{\partial \Delta\rho}\\ &+ \frac{\partial \Delta l}{\partial \theta_1'} \frac{\partial \theta_1'}{\partial \Delta\rho} + \frac{\partial \Delta l}{\partial \theta_2'} \frac{\partial \theta_2'}{\partial \Delta\rho}, \end{aligned} \tag{11}$$

$$\begin{aligned} \frac{\partial \Delta l}{\partial \Delta\theta} = &\frac{\partial \Delta l}{\partial r_1'} \frac{\partial r_1'}{\partial \Delta\theta} + \frac{\partial \Delta l}{\partial r_2'} \frac{\partial r_2'}{\partial \Delta\theta}\\ &+ \frac{\partial \Delta l}{\partial \theta_1'} \frac{\partial \theta_1'}{\partial \Delta\theta} + \frac{\partial \Delta l}{\partial \theta_2'} \frac{\partial \theta_2'}{\partial \Delta\theta}, \end{aligned} \tag{12}$$

where the terms can be derived from Equations (1)–(9).

Equations (10)–(12) can be used to calculate the angular distance change (Δl) according to the initial position of two stars (r_1, r_2, θ_1, θ_2) and the distance and direction of the deviation of the optical center position ($\Delta\rho$, $\Delta\theta$). For any two stars, the impact of the deviation of the optical center on the angular distance change is different at every deviation angle. Since two stars can be located at any position in the FOV, and the deviation angle of the optical center position can also be in any direction, we only consider the range of deviation ($\Delta\rho$). When the angular distance change is limited, the minimum value of all distances of the deviation corresponding to the star pairs at all positions under all deviation angles is taken as the upper limit of the allowable range of deviation. Because Δl has six variables (r_1, r_2, θ_1, θ_2, $\Delta\rho$, $\Delta\theta$), it is complicated to find the upper limit of the allowable range of deviation by Equation (10), so we rely on numerical simulation to find the upper limit.

4. Impact of the Optical Axis Deviation from Numerical Simulation

In this section, we present the results of numerical simulation in two parts. First, we limit the angular distance change between any two stars in the FOV to be less than 1 μas to find the upper limit on the allowable range of deviation. Second, we limit the deviation of optical center position and simulate the distribution of angular distance change under different deviations.

4.1. Finding the Upper Limit on Optical Center Deviation

We set an FOV so that the radius is 0°.22 and the maximal distance difference $g = 1$ μas. We aim to find an upper limit $\Delta\rho_{\max}$ such that if the deviation of optical center position is smaller than it, Δl will not exceed 1 μas. The procedures are outlined below.

1. Choose any two points in the FOV as the star pair, and calculate the distance l.
2. Simulate the deviation of optical center position at different angles and distances. According to Equations (1)–(7), calculate the new coordinates of two stars due to the deviation.
3. Calculate the new distance l' by the new coordinates and the angular distance change Δl between l and l'.

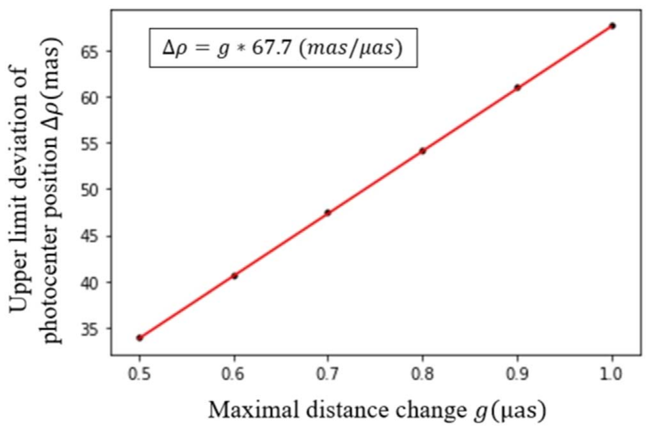

Figure 3. Linear relationship between the deviation of the optical center position and the angular distance change by numerical simulation.

4. Change the angle and increase the distance for the deviation of optical center position and repeat steps (2) and (3) until the angular distance change Δl reaches the given value g. Record the maximum distance of the deviation at the deviation angle.
5. Choose minimum permitted deviation distance of every angle to be the upper limit of the deviation of optical center position for this star pair.
6. Traverse the entire FOV to simulate the two stars anywhere in the considered area. Repeat the above steps and get the corresponding value. Choose minimal permitted deviation distance of every star pair to be the upper limit $\Delta\rho_{\max}$ of the deviation of optical center position for the given value g.

Figure 3 shows that when we limit the angular distance change to 1 μas, the upper limit of the deviation of optical center position in numerical simulation is about 68 mas. In other words, we give such an upper limit, and as long as the deviation of optical center position does not exceed this upper limit, the angular distance will not change more than 1 μas, no matter where the two stars are in the FOV.

According to the numerical simulation, we found that the angular distance is most affected by deviation of the optical center when the two stars are located at the optical center and the edge of the FOV respectively, and the direction of deviation is on the line between the two stars. Substituting this condition into Equations (10)–(12), we can obtain the formula: $\partial\Delta l/\partial\Delta\rho = 1/67.8$ (μas/mas), which is consistent with the simulation result in Figure 3.

In short, for the detection of exoplanets, when the deviation of optical center position is less than 68 mas and the change of angular distance caused by other factors is not considered, the measurement precision of the position can be higher than 1 μas.

4.2. Distribution of Angular Distance Change in Observations

When observing a star, there are generally more than eight reference stars in the FOV (Malbet et al. 2012). The deviation of optical center position will cause offsets to all the nine star positions, resulting in the change of angular distances between star pairs. We took the average of eight pairs of angular distance changes as the impact of the deviation of optical center position, $\Delta l_d = \frac{1}{8}\sum|l_i' - l_i|$. For real observations, the distance between the optical center and the target star is generally within a few arcseconds. The simulation in this paper assumes that the distance between the optical center and the star is within 1″, and there are eight reference stars in the FOV, which are randomly distributed. We simulated the change in angular distance for a given deviation of optical center position and a random deviation angle. Figure 4 displays the distribution of the angular distance change when the deviation of optical center position reaches the set values.

For the real observation, the angular distance change is not only caused by stability of the optical axis, but also due to accuracy of the CCD, optical distortion and other factors that will also affect the angular distance. So the allowable angular distance change caused by the deviation in optical center position should be less than 1 μas. Meanwhile, the actual deviation value is generally less than the set deviation value. Therefore, the distribution of angular distance change will move toward the smaller end.

We took the allowable angular distance changes of 0.3, 0.4 and 0.5 μas as examples and give the ratio of the angular distance changes exceeding the allowable value under different deviations of optical center position in Table 1. We found that as the deviation in optical center position increases, the ratio of the angular distance change exceeding allowable value increases rapidly. According to the actual condition, we can select an appropriate allowable upper limit on the deviation in optical center position, which can ensure that the final angular distance change is no more than 1 μas.

5. Discussion

In addition to the stability of the optical axis, the instrument and optical system will also affect measurement accuracy. For a CCD, the use of micropixel technology is required to achieve the 10^{-5} pixel precision level. For the lens distortion and focal length change, we also propose the following solution to reduce the impacts.

5.1. Reduce the Variation in Distortion

For an optical system, distortion is inevitable and the effect of distortion is not the same at different positions in the FOV (Jin et al. 2013). Due to the space position and attitude of the telescope and the proper motions of the reference stars, the

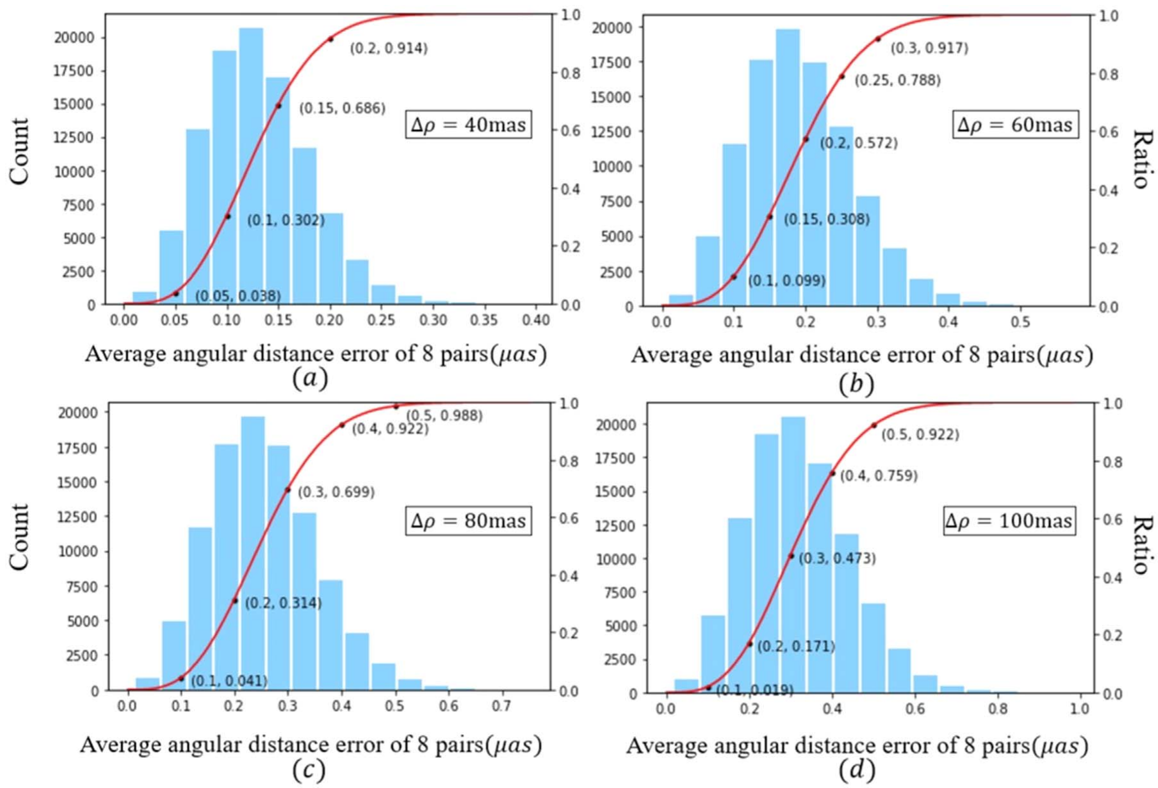

Figure 4. Distribution of the angular distance change with the deviation of optical center position at (a) 40 mas, (b) 60 mas, (c) 80 mas and (d) 100 mas. The red curves in the figure are the cumulative distribution functions. The number of simulations is 100 k.

Table 1
Ratio of the Angular Distance Change Exceeding 0.3, 0.4 and 0.5 μas Under Different Deviations of Optical Center Position

Deviation $\Delta\rho$ (mas)	Ratio		
	>0.3 (μas)	>0.4 (μas)	>0.5 (μas)
40	0.2%	0.0%	0.0%
60	8.3%	0.7%	0.0%
80	30.1%	7.8%	1.2%
100	52.6%	24.1%	7.8%

distribution of reference stars in the FOV will change when observing a target star at different epochs. This position change will cause the image of the reference star to be affected by different distortion effects when passing through the optical system. It will impact the accuracy of the reference star position on the CCD, and eventually lead to an error in angular distance measurement. By mechanical alignment, the impact of space position and attitude change can be avoided, which makes the FOV consistent for each observation. For the impact caused by the reference star's proper motion, we propose that further rotation adjustment of the FOV along the optical axis can be taken to make the position change of all the reference stars in the FOV be reduced in an average sense and reduce impact of the distortion change. We took the following approach to evaluate the effectiveness of reducing this impact.

The impact of the distortion change on the position measurement can be expressed by I,

$$I = \frac{\sum_i^n |\boldsymbol{s}_i - \boldsymbol{s}_i'|\, f_i}{\sum_i^n f_i}, \tag{13}$$

where I is the weighted average of the change in location of the reference stars, n the number of reference stars in the FOV, $\boldsymbol{s}_i$ and $\boldsymbol{s}_i'$ the location of the reference stars in the FOV at the first and current observation respectively, and f_i the weight.

The relation between the distortion (D) of the optical system and the field angle (θ) is $D(\theta) = k_1\theta^3 + k_2\theta^5$ (Jin et al. 2013), so we approximate the weight f_i as the third power of the distance change, $f_i = |\boldsymbol{s}_i - \boldsymbol{s}_i'|^3$. As a result, we have

$$I = \frac{\sum_i^n |\boldsymbol{s}_i - \boldsymbol{s}_i'|^4}{\sum_i^n |\boldsymbol{s}_i - \boldsymbol{s}_i'|^3}. \tag{14}$$

We then define the reduction rate $R = \frac{I' - I}{I} \cdot 100\%$, which represents the impact of the distortion changes before (I) and after (I') rotation. This can be used to evaluate the effectiveness of rotation.

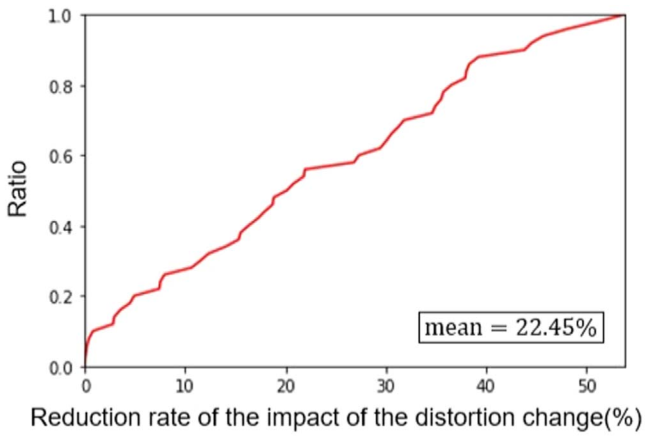

Figure 5. The cumulative distribution function of the maximum reduction rate of the impact of the distortion change by the optical axis rotation.

We selected 50 target stars and a total of 1712 reference stars from a target star list (H.-G. Liu 2015, Private Communication). The average displacement for these reference stars in five years is 56 mas and 90% of the displacements are less than 171 mas. The larger the proper motion of the reference star is, the more it will be affected by non-uniformity and the more the distortion of the optical system will be.

For 50 groups of reference stars, through rotation of the FOV along the optical axis, the cumulative distribution function of the reduction rate of the impact of the distortion changes is plotted in Figure 5. The average reduction rate is about 22%. The rotation angle is on the arcsecond scale, with an average of $44''.7$. The effectiveness of rotation is mainly related to the direction of proper motion, but shows weak dependency on the number and proper motion value of reference stars.

5.2. Monitoring the Change in Focal Length

During the installation of the lens and the operation of the telescope in space, external factors such as temperature will lead to deformation of the lens, which will impact the focal length of the lens. This will cause the star image to defocus on the CCD, resulting in inaccurate location and error in the angular distance measurement.

A collimation system can be added to the telescope to monitor this deviation. The laser irradiates the mirror and the reflected laser irradiates the CCD. If the focal length changes, the image of the laser on the CCD will be defocused, so it can be used to monitor whether the focal length changes.

6. Summary

In this paper, we study the impact of the deviation of optical center position on the angular distance of star pairs. We performed a theoretical analysis of the position of a star on the CCD impacted by the deviation of optical center position and derived the impact on the angular distance between two stars. We discuss the impact from two aspects through numerical simulation.

First, when we limit the distance change between the two stars at any position in the FOV to be less than 1 μas, the upper limit of the deviation of optical center position is about 68 mas. Second, we limit the value of the deviation of optical center position and simulate the angular distance change between the target star and the reference star at the deviation of 40 mas, 60 mas, 80 mas and 100 mas. The proportions with angular distance change exceeding 0.3, 0.4 and 0.5 μas are given as reference.

We also consider other factors that may affect the measurement and give solutions to reduce them. For lens distortion, the telescope can be rotated along the axis to reduce the impact of distortion change. For the change of focal length, a collimating laser can be used to monitor this change.

Acknowledgments

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