THE ASTRONOMICAL JOURNAL, 156:13 (6pp), 2018 July https://doi.org/10.3847/1538-3881/aac518

Link between the VLBI and *Gaia* Reference Frames

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Received 2018 April 1; revised 2018 May 10; accepted 2018 May 13; published 2018 June 14

Abstract

The link between the International Celestial Reference Frame at radio wavelength and the forthcoming *Gaia* optical reference frame is a mandatory task after the completion of the *Gaia* mission. Starting from the provisional reference frame in which *Gaia* astrometric solutions were obtained, we discuss the ways to correct the residual rotation and acceleration effects and investigate three potential options for linking the two frames realized by extragalactic sources. We have estimated the accuracy for the frame alignment assuming different astrometric models of quasar proper motions observed by very long baseline interferometry (VLBI). Using about 500,000 high-precision proper motions of extragalactic sources, the residual rotation of the *Gaia* reference frame is evaluated under 1 μas yr^{-1}. In view of its favorable properties, *Gaia* should be given priority to be considered as the future fundamental reference frame that is consistent with the principle of the International Celestial Reference System. The VLBI reference frame can be linked to *Gaia* based on thousands of common quasars with an accuracy of 10 μas for each axis.

Key words: astrometry – proper motions – quasars: general – reference systems

1. Introduction

The ESA space astrometry mission *Gaia*, launched in late 2013, was designed to make precise measurements of positions and proper motions of both Galactic stars and extragalactic quasars (Gaia Collaboration et al. 2016). Absolute trigonometric parallaxes and multi-epoch photometry and spectroscopy data to uniform precision will be available on schedule in the next data release (DR2) for more than 1 billion sources. The observational principle of *Gaia* stays similar to that of the *Hipparcos* satellite, which utilizes the two widely separated fields of view to scan the celestial sphere in an "along scan" (AL) mode. The raw *Gaia* observations are reduced to an internally consistent and rigid provisional reference frame using the Astrometric Global Iterative Solution (AGIS; Lindegren et al. 2012), but the orientation of the AGIS frame is left essentially arbitrary.

Currently, the astronomical reference system was realized by the very long baseline interferometry (VLBI) observations of distant quasars, and the second version of the International Celestial Reference Frame (ICRF2; Fey et al. 2015) contains more than 3000 selected compact extragalactic sources. The ICRF2 presents a noise floor of about 40 μas and the axes are defined by the coordinates of 295 defining sources and are stable over the period from 1979 to at least 2013 (Lambert 2013). For the alignment between the forthcoming *Gaia* reference frame and the third International Celestial Reference Frame (ICRF3; Jacobs et al. 2014), Bourda et al. (2008) have given details for additional observations focused on 243 optical bright radio sources that are suitable for *Gaia* detection. Orosz & Frey (2013) investigated the optical–radio positional offsets between the ICRF2 catalog and the DR9 of the Sloan Digital Sky Survey. No systematic bias between optical and radio positions was found, but individual sources with large offsets (outliers), which are probably caused due to astrophysical reasons, should be excluded for the *Gaia*-VLBI reference frame link (Zacharias & Zacharias 2014).

The first data release of *Gaia* (Gaia Collaboration et al. 2016; Lindegren et al. 2016) has published more than 1 billion sources brighter than the magnitude of 20.7. For the purpose of calibration, 2191 quasars recognized as the optical counterparts of the ICRF2 sources are separately listed as an auxiliary quasar solution. Careful comparison of the optical and radio positions of these auxiliary quasars shows good consistency between the *Gaia* and ICRF2 catalogs (Mignard et al. 2016). Further comparative study between the ICRF2, Goddard VLBI solution (gsf2016a, potentially to be the ICRF3), and the *Gaia* auxiliary quasar positions revealed significant declination-dependent bias between *Gaia* DR1 and VLBI observations (Liu et al. 2018b). The authors claimed that this feature is mainly attributed to the errors in the southern ICRF2 catalog. When considering the quasar proper motions, different VLBI solutions gave out distinct results for the reference frame link with *Gaia* (Liu et al. 2018a). The above results motivate us to study the link between the reference frames realized by VLBI and *Gaia* observations, aiming to construct a self-consistent, rigid, multiwavelength, and, most importantly, inertial frame that is compatible with the concept of the International Celestial Reference System (ICRS; Feissel & Mignard 1998).

We first review briefly in Section 2 the relations between the dynamical frame FK5 and the first generation of the ICRF (ICRF1), as well as the alignment of the *Hipparcos* to the ICRF1. This helps in understanding the relation between the current ICRF and the future *Gaia* reference frame. In Section 3, we show that *Gaia* is more self-consistent, thus should be considered as a benchmark reference frame, then we discuss in Section 4 three possible approaches to link the VLBI to the *Gaia* reference frame. Section 5 is the discussion on the source variation and optical reference frame realized by a large quantity of Galactic stars. The final conclusions are given in Section 6.

2. From FK5 Reference Frame to the ICRF and the *Hipparcos* Reference Frame

Before the adoption of the ICRS in 1998 (Feissel & Mignard 1998), the fundamental reference system (mean equator and equinox at J2000.0) was materialized by the FK5 catalog consisting of precise positions and proper motions for 1535 bright stars. At the 23rd IAU General Assembly in 1997, the new concept of kinematically nonrotating ICRS, which uses distant extragalactic objects (in radio wavelength) as its new basis, has been authorized to replace the optical FK5 reference frame. For the sake of continuity, the orientation of the ICRS was chosen to be close to the system of the J2000.0 mean equator and equinox defined by the IAU 1976 precession model (Lieske et al. 1977). In the meantime, the *Hipparcos* space mission (ESA 1997) has accomplished (and the *Hipparcos* catalog production was recognized to be) the primary realization of the ICRS at the optical wavelength (Kovalevsky et al. 1997).

Based on the wide-angle measurement property of the *Hipparcos* satellite over the whole sky, the positions and proper motions of *Hipparcos* stars cannot be fixed to an a priori reference frame. The link of the provisional *Hipparcos* catalog, called H37C, to the ICRF is necessary. Due to the magnitude limit of about $V = 12.3$, only one brightest quasar (3C273) was able to be observed by the *Hipparcos* satellite. Therefore, various methods and techniques were adopted to perform the link, including (1) observations of radio stars by VLBI, MERLIN, and VLA; (2) link observations of the *Hubble* telescope with the fine guidance sensors (FGSs); (3) photographic observations of the optical counterparts of extragalactic radio sources; (4) the link using proper motion surveys such as North and South Proper Motion (NPM and SPM) program; and (5) the indirect link by means of Earth orientation parameters (Kovalevsky et al. 1997). The relationship between the H37C and the ICRF is presented by a rigid orientation offset vector $\boldsymbol{\epsilon} = (\epsilon_1, \epsilon_2, \epsilon_3)$ and its changing rate $\boldsymbol{\omega} = (\omega_1, \omega_2, \omega_3)$. Kovalevsky et al. (1997) and Lindegren & Kovalevsky (1995) have carried out two independent syntheses of the results coming from different techniques, and the mean value of the two methods was adopted as the final solution of the link. The orientation and spin vector components are all nominally zero, while the standard errors of the parameters were estimated as 0.6 mas for each component of $\boldsymbol{\epsilon}$ and 0.25 mas yr^{-1} for each component of $\boldsymbol{\omega}$.

3. The *Gaia* Reference Frame

3.1. Gaia *as Fundamental Realization of the ICRS*

From its very principle, *Gaia* is designed to carry out absolute astrometry and the data releases will include simultaneously a set of extragalactic sources used to build an nonrotating frame and a large stellar catalog down to the 20th magnitude. As many authors have claimed, *Gaia* can naturally build a self-consistent reference frame (e.g., Perryman et al. 2014; Gaia Collaboration et al. 2016; Lindegren et al. 2016; Mignard et al. 2016) because of the well-designed mode of the astrometric observations in space. No external model is necessarily input: the only possible effect that gives rise to the uncertainty of reference frame is the satellite orbital determination. On the other hand, the ground-based VLBI, which is highly successful in establishing celestial and terrestrial reference frames, depends on the rotation of the Earth, International Terrestrial Reference Frame (ITRF) data, precession–nutation model, atmospheric model, strategy of observations, constraints in data analysis, and some occasional data outliers (Malkin 2015). Furthermore, we note that possible systematics in VLBI catalogs has been discovered (Liu et al. 2018b); therefore, it is preferable to take *Gaia* as the primary reference frame in the era of microarcsecond astrometry. From the historical point of view, the fundamental reference frame can return to optical domain thanks to the new *Gaia* mission.

3.2. *From the* Gaia *Provisional Reference Frame to the Final* Gaia *Reference Frame*

Follow the prescriptions of the ICRS (Feissel & Mignard 1998), we only consider extragalactic sources in the following discussion. Let us start from the the provisional reference frame, i.e., the AGIS frame (Lindegren et al. 2012) in which the full astrometric parameters of more than 500,000 extragalactic sources (most of them being quasars) will be obtained. This frame should first be adjusted to be an inertial reference frame and then aligned with the ICRF2 (Fey et al. 2015) or ICRF3 (Jacobs et al. 2014) for continuity.

In the AGIS frame, all sources should have apparent motions caused by the residual rotation and the galactic aberration effect (e.g., Liu et al. 2012; Titov & Lambert 2013; Malkin 2014), hence the spatial velocities of quasars can be written as

$$\boldsymbol{v}_{gaia} = \boldsymbol{v}_{\boldsymbol{\omega},gaia} + \boldsymbol{v}_{\boldsymbol{a},gaia} + \boldsymbol{v}_{\mathrm{intrinsic}} + \mathrm{noise}, \tag{1}$$

where the first two terms on the right-hand side, $\boldsymbol{v}_{\omega}$ and $\boldsymbol{v}_{a}$, represent the motions induced by the global spin and the galactic aberration, respectively:

$$\begin{aligned} \boldsymbol{v}_{\boldsymbol{\omega},gaia} &= \boldsymbol{u} \times \boldsymbol{\omega}_{gaia} \\ \boldsymbol{v}_{\boldsymbol{a},gaia} &= \boldsymbol{u} \times (\boldsymbol{a}_{gaia} \times \boldsymbol{u}). \end{aligned} \tag{2}$$

In the above equations, the unit vector $\boldsymbol{u} = (\cos\alpha\cos\delta, \sin\alpha\cos\delta, \sin\delta)$ is in the direction of a source, $\boldsymbol{\omega}_{gaia} = (\omega_1, \omega_2, \omega_3)$ is the spin vector for the provisional reference frame, and $\boldsymbol{a}_{gaia} = (a_1, a_2, a_3)$ is the acceleration of the solar system barycenter expressed in the unit of angular proper motions. Theoretically, the acceleration $\boldsymbol{a}$ points to the direction of the Galactic center ($\alpha_{\mathrm{G}} \simeq 267°$, $\delta_{\mathrm{G}} \simeq -29°$) whose amplitude lies in the range of 4–6 μas yr^{-1}, and it is the only significant effect that can be detected by *Gaia* with μas yr^{-1} accuracy (Bachchan et al. 2016). The smallness of the angles ($<10^{-10}$ rad) allows us to apply small-angle approximations safely in the following study (Lindegren et al. 2012).

The last two terms in Equation (1) are velocities resulting from source intrinsic motion and random noise of astrometric observations. Taris et al. (2018) found that the variability of extragalactic sources in optical wavelength causes apparent proper motions; therefore, it has an impact on the *Gaia* reference frame. These specific sources should be excluded in the final alignment of the reference frames. In the following, we first neglect the intrinsic velocity term and discuss this effect in Section 5.1.

Given the celestial coordinates (α, δ) of a source under consideration, we can project the 3D velocity in Equation (1) on the local tangential plane. This gives the apparent proper motions of the sources:

$$\begin{aligned} \mu^{*}_{\alpha,gaia} &= \boldsymbol{v}_{gaia} \cdot \boldsymbol{p} = +\boldsymbol{q} \cdot \boldsymbol{\omega}_{gaia} + \boldsymbol{p} \cdot \boldsymbol{a}_{gaia} + \mathrm{noise} \\ \mu_{\delta,gaia} &= \boldsymbol{v}_{gaia} \cdot \boldsymbol{q} = -\boldsymbol{p} \cdot \boldsymbol{\omega}_{gaia} + \boldsymbol{q} \cdot \boldsymbol{a}_{gaia} + \mathrm{noise}, \end{aligned} \tag{3}$$

where $\boldsymbol{p} = (-\sin\alpha, \cos\alpha, 0)$ and $\boldsymbol{q} = (-\cos\alpha\sin\delta, -\sin\alpha\sin\delta, \sin\delta)$ are the unit vectors toward the directions of increasing R. A. and decl. Note that unit vectors $\boldsymbol{p}$, $\boldsymbol{q}$, and $\boldsymbol{u}$ make up a local rectangular coordinate system centered at the source. The spin vector and the solar acceleration are determined simultaneously by least squares fits using the linear condition of Equation (3) and proper motions of more than 500,000 quasars observed by *Gaia*. From simulations, the accuracy of the spin and acceleration vector components are on the order of 0.2–0.4 μas yr^{-1} (Perryman et al. 2014; Bachchan et al. 2016). The proper motions introduced by $\boldsymbol{\omega}_{gaia}$ and $\boldsymbol{a}_{gaia}$ are considered a systematic part in the original *Gaia* provisional proper motions and should be subtracted. After that, the remaining proper motions consist of only noises that are completely random without any global rotation and streaming pattern. Up to this stage, the inertial reference frame of *Gaia* has been established.

At any chosen standard epoch T for the purpose of linking inertial reference frames, the positions of *Gaia* extragalactic sources are given by

$$\boldsymbol{u}_{gaia}(T) = \boldsymbol{u}_{gaia}(t) - (\boldsymbol{v}_{\omega,gaia} + \boldsymbol{v}_{\boldsymbol{a},gaia})(t - T), \tag{4}$$

in which t stands for the mean observational epoch for a source.

4. Linking the VLBI Reference Frame to the *Gaia* Reference Frame

Now we turn to the ICRF realized by the VLBI techniques. At present, the apparent motion induced by solar acceleration was not considered for the quasars. Liu et al. (2012) have studied the global feature of the galactic aberration and pointed out that the acceleration introduces tiny global rotation depending on the source distribution. Using VLBI observations, the acceleration of the Sun has been determined, e.g., by fitting series of source coordinates (Titov & Lambert 2013), by solving the acceleration as a global parameter (Xu et al. 2012), by means of the Earth scale factor (Titov & Krásná 2018). To construct an inertial VLBI reference frame, the acceleration effect should be eliminated: this can be done in several ways depending on the astrometric model of radio source proper motions.

4.1. Method 1

Traditionally, the peculiar motion of the extragalactic sources are considered as zero and no proper motion concept is introduced in VLBI data reduction. To this extent the apparent space motions of extragalactic sources are simply attributed to the acceleration of the Sun:

$$\boldsymbol{v}_{\rm vlbi} = \boldsymbol{v}_{\boldsymbol{a},\rm vlbi} + \text{noise}. \tag{5}$$

Since the aberration is a small astrometric effect that can be totally eliminated, the positions of VLBI sources at an epoch T (same as the epoch in Equation (4)) should be

$$\boldsymbol{u}_{\rm vlbi}(T) = \boldsymbol{u}_{\rm vlbi}(t) - \boldsymbol{v}_{\boldsymbol{a},\rm vlbi}(t - T). \tag{6}$$

After this stage, the VLBI reference frame can be considered as an inertial frame parallel to *Gaia* after adjustments. For the value of the acceleration used in Equation (6), one can choose a theoretical value of $\boldsymbol{a}$ whose error is under 1 μas yr^{-1} (Malkin 2014); however, we prefer to adopt the fitted value of $\boldsymbol{a}_{gaia}$ in Equation (1). This is compatible with the principle that *Gaia* observations should be the basis of the reference frame.

To link the VLBI to the *Gaia* reference frame (both are inertial without global rotation) at the same epoch T, only the orientational offset $\boldsymbol{\epsilon}(T)$ between the two reference frames is necessarily calculated:

$$\Delta\boldsymbol{u}(T) = \boldsymbol{u}_{gaia}(T) - \boldsymbol{u}_{\rm vlbi}(T) = \boldsymbol{\epsilon}(T) \times \boldsymbol{u}_{\rm vlbi}. \tag{7}$$

Projecting $\Delta\boldsymbol{u}(T)$ in the direction of $\boldsymbol{p}$ and $\boldsymbol{q}$, we have the condition equations

$$\begin{aligned} \Delta\alpha^*(T) &= \Delta\boldsymbol{u}(T)\cdot\boldsymbol{p} = +\boldsymbol{q}\cdot\boldsymbol{\epsilon}(T) \\ \Delta\delta(T) &= \Delta\boldsymbol{u}(T)\cdot\boldsymbol{q} = -\boldsymbol{p}\cdot\boldsymbol{\epsilon}(T) \end{aligned} \tag{8}$$

that connect the difference in source coordinates (*Gaia*−VLBI) and the orientation offset $\boldsymbol{\epsilon}(T)$. We can obtain the vector $\boldsymbol{\epsilon}(T)$ by using least squares and common sources in the *Gaia* and VLBI catalogs. Here, an epoch near J2015.0 (the approximate median epoch of the *Gaia* astrometric solution) is favorable for the the purpose of minimizing the correlation between the estimated parameters (Lindegren et al. 2012).

4.2. Method 2

As VLBI is now able to detect proper motions of quasars either by means of time series of source coordinates or global solutions (Liu et al. 2018a), the inertial reference frame materialized by radio sources can be established in the same way as *Gaia*. The velocities of quasars in the provisional VLBI reference frame are thus modeled in a similar way to Equation (1):

$$\boldsymbol{v}_{\rm vlbi} = \boldsymbol{v}_{\omega,\rm vlbi} + \boldsymbol{v}_{\boldsymbol{a},\rm vlbi} + \boldsymbol{v}_{\rm structure} + \text{noise}. \tag{9}$$

Here, the term of source structure is complicated and sources with strong evidence of structure variations should be excluded. For the remaining source we first use all available VLBI data to estimate the spin and acceleration effects in the VLBI reference frame with the following equations:

$$\begin{aligned} \mu^*_{\alpha,\rm vlbi} &= \boldsymbol{v}_{\rm vlbi}\cdot\boldsymbol{p} = +\boldsymbol{q}\cdot\boldsymbol{\omega}_{\rm vlbi} + \boldsymbol{p}\cdot\boldsymbol{a}_{\rm vlbi} + \text{noise} \\ \mu_{\delta,\rm vlbi} &= \boldsymbol{v}_{\rm vlbi}\cdot\boldsymbol{q} = -\boldsymbol{p}\cdot\boldsymbol{\omega}_{\rm vlbi} + \boldsymbol{q}\cdot\boldsymbol{a}_{\rm vlbi} + \text{noise}. \end{aligned} \tag{10}$$

Then the global spin and acceleration induced velocities are removed from the observed apparent motions. At a chosen standard epoch T, the source positions that realize an inertial reference frame are such that

$$\boldsymbol{u}_{\rm vlbi}(T) = \boldsymbol{u}_{\rm vlbi}(t) - (\boldsymbol{v}_{\omega,\rm vlbi} + \boldsymbol{v}_{\boldsymbol{a},\rm vlbi})(t - T). \tag{11}$$

The formulations for the orientation offset are the same as Equations (7) and (8).

In this method, apparent proper motions of quasars are taken into account and the provisional VLBI reference frame is first corrected for the global rotation and acceleration effects to build an inertial radio reference frame. The above procedure is equivalent to the *Gaia* side as in Section 3.2. From an observational point of view, proper motions of quasars are observable with high, precise VLBI astrometry, so that this method may be more reasonable in practice.

4.3. Method 3

In the third method, we use the differential proper motions of quasars between *Gaia* and VLBI to eliminate the global spin between the two reference frames. Limited by VLBI observations, only a small number (several thousands) of common sources in radio and optical wavelengths will be included. The differential apparent motion between common sources in the two systems are derived by subtracting Equation (9) from Equation (1):

$$\Delta \boldsymbol{v} = \boldsymbol{v}_{gaia} - \boldsymbol{v}_{\mathrm{vlbi}} = \boldsymbol{v}_{\Delta\omega} + \boldsymbol{v}_{\Delta \boldsymbol{a}} + \Delta\mathrm{noise}, \tag{12}$$

where the structure and source variation effects are neglected. The three terms in the right-hand side are quasar motions due to relative rotation, relative acceleration, and noise. The second term $\boldsymbol{v}_{\Delta \boldsymbol{a}}$ should vanish ($\Delta \boldsymbol{a} = 0$) as the acceleration of the Sun is a physical effect having the same contribution to VLBI and *Gaia* derived source motions. Accordingly, we obtain the condition equations for deriving the relative spin:

$$\begin{aligned} \Delta\mu_{\alpha}^{*} &= \Delta \boldsymbol{v} \cdot \boldsymbol{p} = +\boldsymbol{q} \cdot \Delta\boldsymbol{\omega} \\ \Delta\mu_{\delta} &= \Delta \boldsymbol{v} \cdot \boldsymbol{q} = -\boldsymbol{p} \cdot \Delta\boldsymbol{\omega}. \end{aligned} \tag{13}$$

The components of $\Delta\boldsymbol{\omega}$ are determined by least squares fits of differential proper motions of thousands of quasars to the above equations.

According to Equation (9) the observed velocity of VLBI sources can be written as

$$\boldsymbol{v}_{\mathrm{vlbi}} = \boldsymbol{v}_{\boldsymbol{\omega}_{gaia}} + \boldsymbol{v}_{\Delta\omega} + \boldsymbol{v}_{\boldsymbol{a}_{gaia}} + \mathrm{noise}. \tag{14}$$

Here, *Gaia* is considered as the basic inertial reference frame: the vectors $\boldsymbol{\omega}_{gaia}$ and $\boldsymbol{a}_{gaia}$ (used to systematic quasar proper motions) are derived using *Gaia* data alone (See Section 3.2). On the other hand, the differential rotation vector $\Delta\omega$ is estimated from the duplicate source in both *Gaia* and VLBI catalogs. Once these systematic motions are excluded, the VLBI inertial reference frame can be constructed. In a further step, the radio source positions for linking to the *Gaia* reference frame at T are such that

$$\begin{aligned} \boldsymbol{u}_{\mathrm{vlbi}}(T) &= \boldsymbol{u}_{\mathrm{vlbi}}(t) - (\boldsymbol{v}_{\omega_{\mathrm{vlbi}}} + \boldsymbol{v}_{\boldsymbol{a},\mathrm{vlbi}})(t - T) \\ &= \boldsymbol{u}_{\mathrm{vlbi}}(t) - (\boldsymbol{v}_{\omega_{gaia}} + \boldsymbol{v}_{\Delta\omega} + \boldsymbol{v}_{\boldsymbol{a},\mathrm{vlbi}})(t - T). \end{aligned} \tag{15}$$

Consequently, the positional differences between *Gaia* and VLBI source at epoch T are

$$\begin{aligned} \Delta\boldsymbol{u}(T) &= \boldsymbol{u}_{gaia}(T) - \boldsymbol{u}_{\mathrm{vlbi}}(T) \\ &= \boldsymbol{u}_{gaia}(t) - \boldsymbol{u}_{\mathrm{vlbi}}(t) + \boldsymbol{v}_{\Delta\omega}(t - T). \end{aligned} \tag{16}$$

The components in R.A. and decl. are then derived as

$$\begin{aligned} \Delta\alpha^{*}(T) &= [\boldsymbol{u}_{gaia}(t) - \boldsymbol{u}_{\mathrm{vlbi}}(t)] \cdot \boldsymbol{p} + \boldsymbol{q} \cdot \Delta\boldsymbol{\omega}(t - T) \\ &= +\boldsymbol{q} \cdot \boldsymbol{\epsilon}(T) \\ \Delta\delta(T) &= [\boldsymbol{u}_{gaia}(t) - \boldsymbol{u}_{\mathrm{vlbi}}(t)] \cdot \boldsymbol{q} - \boldsymbol{p} \cdot \Delta\boldsymbol{\omega}(t - T) \\ &= -\boldsymbol{p} \cdot \boldsymbol{\epsilon}(T). \end{aligned} \tag{17}$$

These equations of condition are used to obtain orientation offsets $\boldsymbol{\epsilon}(T)$ by least squares fits.

4.4. Error Estimates for the Link between the Reference Frames

In the above three methods, the final vector of the orientation offset $\boldsymbol{\epsilon}(T)$ is used to rotate the VLBI reference frame to the new *Gaia* frame by adding $+\boldsymbol{q} \cdot \boldsymbol{\epsilon}(T)$ and $-\boldsymbol{p} \cdot \boldsymbol{\epsilon}(T)$ to the R.A. and decl. of radio sources. As a result, the orientation of the VLBI reference frame will coincide exactly with *Gaia*, and this frame can be regarded as a realization of the inertial reference system at radio wavelength. The accuracy of $\boldsymbol{\epsilon}(T)$ is excepted to be the same for all of the three mentioned methods since they use the same set of common extragalactic source. Based on the analysis of Liu et al. (2018b) employing the gsf2016a VLBI global solution and the *Gaia* DR1 auxiliary quasar solution, the global orientation offset between the VLBI and *Gaia* frames is smaller than 100 μas at the epoch of $T = \mathrm{J}2015.0$, with an accuracy of about 20 μas for each component of $\boldsymbol{\epsilon}(T)$. This value will certainly be improved in the next few months with the application of the second data release of *Gaia* (DR2) and the upgraded VLBI catalog such as the ICRF3.

Now we return to the degree of inertia of both reference frames that are related to the formal uncertainties of the residual rotation vector $\boldsymbol{\omega}$ and the acceleration vector $\boldsymbol{a}$, both of them being fitted using the least squares method but with different sets of quasars. For *Gaia*, as we stated in Section 3.2, the precision of the residual rotation and acceleration is as extremely high as tenths of μas yr^{-1}.

The situations for the VLBI reference frame are, however, not the same for each method depending on different astrometric modeling and data sets. In the first method, only the acceleration effect is taken into account for the VLBI quasar motions. The two options for choosing $\boldsymbol{v}_{\boldsymbol{a},\mathrm{vlbi}}$ (theoretical value or $\boldsymbol{v}_{\boldsymbol{a},gaia}$) to eliminate this effect has a similar uncertainty of several tenth of μas yr^{-1}; therefore, in this method the accuracy of the *Gaia* and VLBI reference frames is homogeneous.

In the second method, we first correct the systematic effects in the radio source motions to transform the provisional VLBI reference frame to an inertial one. The residual rotation and acceleration estimated using VLBI data are not necessarily consistent with *Gaia*'s results since they are solved in completely different frequency domains and observational techniques. Recently, Liu et al. (2018a) analyzed the coordinate time series of quasars provided by the International VLBI Service for Geodesy & Astrometry (IVS). The fitted linear proper motions are used in a further step to estimate the global spin and acceleration effects in the reference frames realized by these sources. Analyses were performed using two data sets (722 sources from Goddard Space Flight Center and 777 sources from Paris Observatory) after rejecting outlier data. The results showed that the formal uncertainties of $\boldsymbol{\omega}_{\mathrm{vlbi}}$ and $\boldsymbol{a}_{\mathrm{vlbi}}$ are about 0.1 μas yr^{-1}, which are slightly better than those from *Gaia* thanks to the very high accuracy of VLBI astrometry (although the number of sources is much smaller). Nevertheless, we address that the results from different IVS analysis centers are not consistent within the standard errors, but in the future official data release of source coordinate series as well as the standard procedure for estimating source proper motions would be helpful to solve this problem.[1]

For the last method about differential quasar motions between *Gaia* and VLBI, Liu et al. (2018a) have carried out the calculation based on IVS data and the simulated *Gaia* proper motions for more than 600 common sources. The uncertainties of $\Delta\boldsymbol{\omega}$ that lie in the interval 2–3 μas yr^{-1} depend on the data sets used in the analysis. Such a magnitude is too

[1] The ICRF3 will be generated as a global solution, as was the ICRF2, and there will not be any official VLBI time series corresponding to ICRF3. One has to use some unofficial series from one or more of the IVS analysis centers, which may or may not agree with ICRF3 after solving for the spin and the galactic aberration.

large compared to the predicted stability or inertia of both the *Gaia* and VLBI reference frame due to a too limited number of common sources; therefore, this method should be used with caution.

5. Discussion

5.1. Source Variation in Optical and Radio Bandpasses

Optical monitoring of quasars indicates that the changes in morphology and displacements of photocenters of sources (for which the timescale ranges from days to years) could be influenced by intrinsic or extrinsic physical processes. Such displacements will be responsible for non-negligible uncertainty (about 50 μas) in the reference frame link between *Gaia* and VLBI (Taris et al. 2018). Moreover, the successive positional offsets of extragalactic sources in the optical band will have a considerable contribution to proper motions (the term $\boldsymbol{v}_{\boldsymbol{a},\mathrm{intrinsic}}$ in Equation (3)); consequently, the fitted rotation vector $\boldsymbol{v}_{\omega,gaia}$ and acceleration vector $\boldsymbol{v}_{\boldsymbol{a},gaia}$ are contaminated. Meanwhile, one will have more significant errors of these parameters than predicted. Therefore, these sources should be carefully studied and excluded in advance when performing the link.

In the radio domain, to get a clean set of extragalactic sources for aligning the VLBI frame to the *Gaia*, these sources need to be monitored in order to control their positional stability and accuracy, as well as to detect possible proper motions resulting from the variation of their VLBI structures. In order to achieve these goals, more VLBI networks and higher frequency observations are applied (Lanyi et al. 2010). At the higher frequency, the bases of radio jets are closer to the optical emission region, hence the core-shift effects are reduced. Gontier et al. (2001) studied the time series of the source coordinates using statistical approaches and have shown clearly the effect of source structure on the stability of the VLBI reference frame. This explains why we should exclude sources with evident structure variations. The forthcoming ICRF3 will surely develop such a clean set of quasars for linking the reference frames.

5.2. The Solar Acceleration Effect in the Gaia *Stellar Reference Frame*

Besides the primary extragalactic source reference frame based on the global zero-proper-motion constraint, one will have a high-density reference frame materialized by galactic stars detected by *Gaia*. The second data release of *Gaia* will publish the full astrometric parameters for more than 1.3 billion sources with a limiting brightness of $G = 21$ mag. The proper motion uncertainties are estimated to be on the order of 60 μas yr^{-1} for $G < 15$ mag, 200 μas yr^{-1} for $G < 17$ mag, and 1200 μas yr^{-1} for $G < 20$ mag.[2]

Since the optical reference frame also has to be established at the barycenter of the solar system, we must consider the contribution of the galactic aberration. Although the origin is the same (i.e., the acceleration of the Sun around the Galactic center), its systematic effect on the Milky Way stars seems more complicated than on the extragalactic sources. To remove the aberration effect from observed proper motions, the stellar acceleration of themselves should also be included (Kovalevsky 2003; Liu et al. 2013). Taking the simplified rotation curve of the Milky Way as the kinematic model of stars in circular orbits, the largest aberration effect was estimated to be 150 μas yr^{-1} near the Galactic center whose importance is only marginal considering the large distance and faint magnitudes of stars. We also note that the detection with high reliability of stellar acceleration stays very difficult; consequently, it is preferable to leave the aberration effect in the stellar proper motions. For the nearby stars with extremely high accurate proper motion measurements (e.g., $\sigma_{\mathrm{pm}} < 5$ μas yr^{-1}), the aberration effect from the well-determined solar acceleration should be corrected. The final step to construct the stellar reference frame, which serves as the practical standard in optical astrometric observations, is to apply the global rotation $-\boldsymbol{\omega}_{gaia}$ (obtained in Equation (3)) to all of the stellar proper motions.

6. Concluding Remarks

In this paper, we have discussed the principle of the *Gaia* and VLBI reference frames realized by distant extragalactic sources. Three methods are proposed for linking the VLBI to the *Gaia* reference frame. The crucial point is that we regard the new *Gaia* reference frame (adjusted by removing residual rotation and acceleration effects) as an inertial, self-consistent, and highly accurate frame and it will be the fundamental reference frame in the future. The VLBI reference frame should be linked to *Gaia* after removing necessary systematic effects in which the galactic aberration is the most important term.

The first method is recommended by previous works such as Mignard (2012), Perryman et al. (2014), and Lindegren et al. (2016) where the VLBI frame plays the base role. In the second method, the present VLBI reference frame is also considered as a provisional frame in which proper motions of sources are obtained. The rotation and acceleration effects are first eliminated from the VLBI system itself. In the next step, the adjusted VLBI frame is linked to the inertial *Gaia* frame at a specific epoch *T*. We have shown that the accuracy of the link is comparable to the first one but it has more clear physical meaning: both *Gaia* and VLBI are adjusted (with similar approaches) to be inertial frames, then the link of the orientation is performed. The last method makes use of the differential proper motions of quasars. The estimated precision of the link is 10 times worse than the previous cases. After linking to the *Gaia*, possible systematic errors (e.g., reflected by global feature $\Delta\boldsymbol{\omega}$ or by individual discrepancies in source positions and proper motions) in the VLBI system can be externally checked and eliminated such that VLBI can serve better the purpose of monitoring the Earth's rotation for the new theories of precession, nutation, and polar motion.

This work is funded by the National Natural Science Foundation of China (NSFC) under grant Nos. 11473013 and 11303018. This work is also supported by the Fundamental Research Funds for the Central Universities under grant No. 020114380023. This work is also supported by the Fundamental Research Funds for the Central Universities under grant No. 020114380023.

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