

Measurement of Earth’s Nutation by VLBI: Direct Estimates from VLBI Delays and a Discussion on the Error

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Abstract Accurate measurements of the Earth nutation by VLBI provide insights into the deformability of and the coupling mechanisms at the core-mantle and core-inner core boundaries. We propose here to adjust nutation amplitudes directly to VLBI delays, as opposed to the traditional method consisting of fitting nutation amplitudes to time-domain nutation series. However, the complexity of the VLBI analysis chain makes the formal error on the parameters somehow obscure and disconnected from a realistic error based on, e.g., empirical tests of robustness and errors on models. In this work, we address some striking differences between formal and empirical errors.

Keywords Earth rotation, VLBI

1 Introduction

Very Long Baseline Interferometry (VLBI) is the only technique that gives access to nutation and precession, that is, the variable orientation of the Earth’s figure axis with respect to space. The amplitude of nutation depends on (i) the amplitude of the external gravitational potential arising from the Moon and the Sun, and (ii) the Earth deformational response to this potential. The latter can be expressed as a function of

parameters describing the Earth’s structure and rheology (e.g., flattening of the various layers, anelasticity coefficients, coupling constants). An accurate measurement of nutations allows therefore to perform geophysical studies related to the Earth interior (see, e.g., [15, 10, 8, 13, 14, 4] and references therein).

Mathews et al. ([10]), who published in 2002 the current conventional nutation model known as MHB 2000 or IAU 2000A, adjusted the so-called basic Earth parameters using an indirect method consisting of two steps: (1) fitting a finite number of amplitudes to nutation time series, and (2) fitting the geophysical parameters to the set of amplitudes. Implicitly, this method needs also the preliminary reduction of VLBI delays to obtain nutation time series, constituting a step (0). Later, [8] used a Bayesian approach to fitting directly the geophysical parameters in the time domain to the nutation time series, treating therefore the steps (1) and (2) of [10] together but still having no control on the nutation data themselves except a classical outlier elimination and a recalibration of the errors (see also [13, 14, 6]). Here, we propose to test a so-called ‘direct’ approach consisting of fitting directly the nutation amplitudes to VLBI delays, as a shortcut of steps (0) and (1). Such an approach avoids multiplying least-squares computations and ensures a rigorous error propagation from time delays to parameters. Beyond the feasibility of such an approach, we want to address the question of the precision of its results.

2 Analysis and Results

We processed all VLBI sessions between 1979 and 2018 with the Calc/Solve geodetic VLBI analysis soft-

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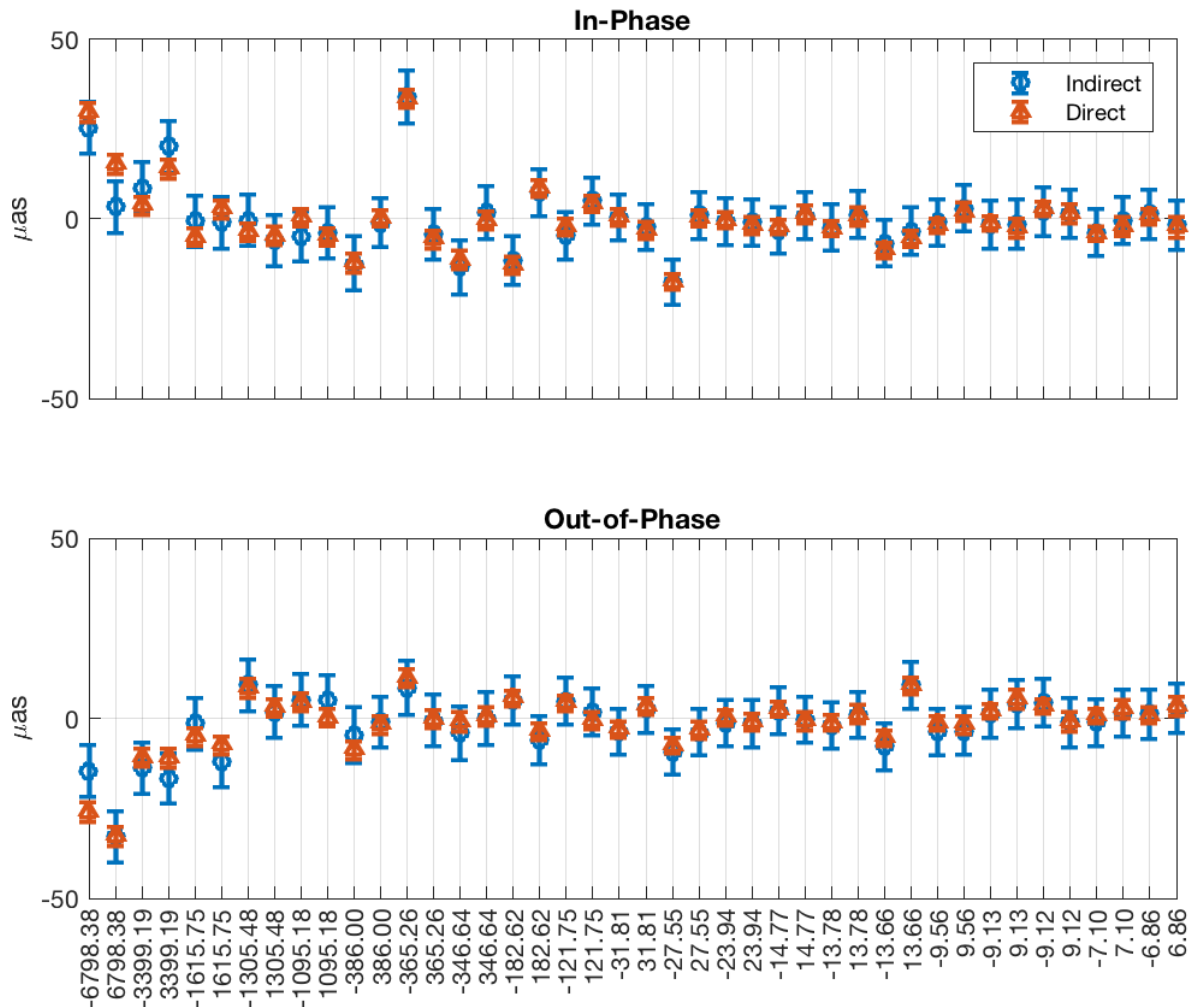


Fig. 1 The corrections to MHB 2000 obtained by the direct and the indirect approaches.

ware package using a standard configuration as described in the Paris Observatory analysis center Web site at <http://ivsopar.obspm.fr/24h>. In the so-called indirect approach, we estimated session-wise nutation offsets to MHB 2000. Then, we estimated the correction to the MHB 2000 amplitudes of the 42 periodic nutation terms whose periods are listed in [10] by applying simple weighted least-squares to the time series. Weights were taken as the inverse of the squared formal error. We also adjusted iteratively a noise floor and a scale factor to the formal errors (see [7, 14, 6]). In the so-called direct approach, we switched off the session-wise nutation estimates and, instead, added partial derivatives of the delay with respect to the am-

plitudes of the same 42 periodic nutation terms. The obtained amplitudes (corrections to MHB 2000) are shown in orange in Figure 1, superimposed onto the amplitudes obtained by the indirect approach. One can see that the amplitudes obtained by the two approaches are fairly consistent, except for the longest periods (18.6 and 9.3-yr) for which the amplitudes are likely more sensitive to the sparse data of the early VLBI years (typically before 1984). One can see also that the error bars of the direct solution are much smaller than the ones of the indirect solution by a factor of about 3 to 4. In both approaches, however, the formal errors are homogeneous across the periods: the uncertainty on the 18.6-yr is only very slightly larger than the one of

the weekly terms. Correlations between nutation amplitudes are largely the same in the two approaches. Both direct and indirect solutions returned a postfit rms of 26 ps and a chi-squared per degree of freedom of 0.94 indicating no further systematics. These results demonstrate that an empirical modeling of the nutations directly adjusted on VLBI delays is feasible.

Now, should we conclude that we have a much more precise result using the direct approach? There are at least two reasons for which one should take these small error bars with great care. The first reason is the existence of external contributions to the nutation that can be considered as unknown. The main one is the contribution of the atmosphere to the diurnal nutation that shows inconsistent values from one model to another (see, e.g., [1, 16, 17, 9]). As an example, we used the output of the two well-known global circulation models from NCEP/NCAR Reanalysis and ECMWF ERA Interim to estimate the atmospheric contribution to some nutations (Figure 2) and we showed that the two models do not deliver the same message. E.g., for the annual retrograde nutation, NCEP/NCAR and ECMWF differ by 100% at the level of 0.1 mas, which is several times the amplitude of the correction to this nutation as shown in Figure 1. As a consequence, one cannot consider seriously the VLBI estimate of the amplitude at better than 0.1 mas, although the formal error is ridiculously small: of a few 0.001 mas. This explains why [10] did not consider the annual nutation for constraining the computation of the basic Earth parameters.

The next reason for which one must consider the formal error with care is the robustness of the estimates. Indeed, the nutation amplitudes were obtained with a certain analysis configuration; but what would happen if one modifies this configuration, even slightly? [6] has derived some empirical errors by comparing the amplitudes of nutation adjusted on various nutation series computed by several IVS analysis centers and showed that the inconsistencies between series could be as large as several tens of μas and very inhomogeneous across the frequencies.

Another possible test of stability is similar to what was used to recalibrate the errors on radio source positions in the ICRF2 and ICRF3 works [5, 2]. It consists of splitting our session list into two groups of approximately the same number of observations. Different networks like R1/R4 and NEOS/CORE were also separated into different session lists. Although the number

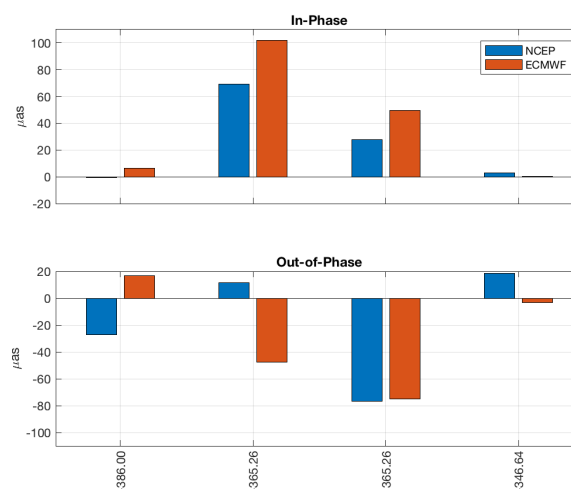


Fig. 2 The contribution of the diurnal variations in the atmospheric angular momentum to the nutation.

of observations is basically halved, the time domain information is not deeply changed with respect to the initial session list, especially for long periods. It seems therefore obvious that one should obtain results close to those obtained with the full session list. Then, we estimated the nutation amplitudes from the two session lists and computed the difference. This difference, that should be zero if the two session lists provided exactly the same result as the full session list, provides an external error characterizing the robustness of the amplitudes. We superimposed this external error onto the previous results of the direct and indirect approach in Figure 3. Interestingly, these external errors are much less homogeneous than the formal errors. For some unclear reasons, some nutation appear as very robust (external error within a few μas) while others are very unstable (external error up to 20 μas for the monthly and the semi-annual terms). The instability of some terms raises here the possibility of some unmodeled effect in the VLBI data reduction model like, e.g., a contribution of the subdaily polar motion ([12]).

3 Conclusion

This paper reports on the first step of a longer study currently undertaken at Paris Observatory in which we want to reestimate the basic Earth parameters of

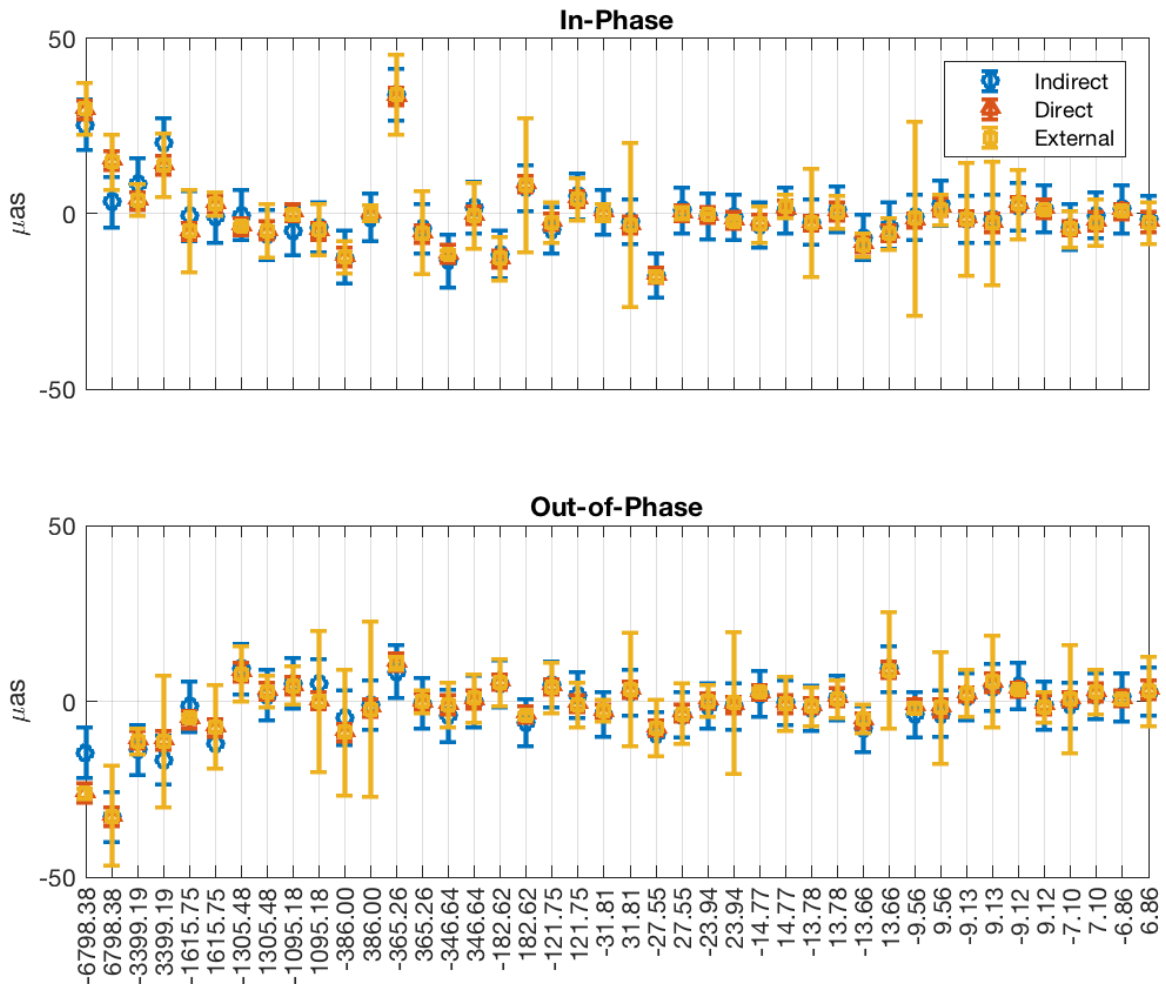


Fig. 3 Same as Figure 1 but with the external error superimposed.

[10] using several approaches (see [11, 18]). The fully-implemented direct approach consists of estimating the basic Earth parameters directly on VLBI delays ([11]), as opposed to a multi-step method used in various previous studies based on nutation time series ([10, 8, 13, 14]). Nevertheless, we aim at pointing out that there are numerous traps due to misinterpreted errors at various stages of the computation. Here, we showed that the direct approach is consistent with the indirect one for the estimation of nutation amplitude. But the formal errors on the amplitudes are somehow meaningless, or, at least, should be complemented by other errors arising from unknown contribution (e.g., atmosphere, oceans, correlation with the forced-free motion associated with the free core nutation, sensitivity to the analysis configuration and the observation

list) obtained from, e.g., stability tests. These external errors could be used as weighting factors in further scientific exploitation of the nutation amplitudes.

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